THE LEARNING TYPE OF MEAN AND MEDIAN HYBRID FILTERS

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ABSTRACT
In this paper, a new adaptive filter, called learning type of mean and median hybrid (LMMH) filters, is introduced. This filter is a combination of FIR filtering and order statistics (OS) filtering for removal all kinds of distributed noise. LMMH filter is regarded as the extension of MMH filters which can’t be learned. On the other hand, LMMH filters can be optimized by using a priori information on input signal. A procedure for designing an optimal LMMH filters under the mean square error criterion has been developed. Experimental results show that the performances of the optimal LMMH filter are superior to those of the Wiener filter and the OS filter, for signal corrupted by from short- to long-tailed distributed noise.

1. INTRODUCTION
In many signal processing applications, the suppression of the noise in an input signal sequence can be achieved by linear filtering\cite{1}. Linear filters are well-known to be suitable for eliminating Gaussian noise. In some situations, however, linear filtering is inadequate for signal restoration. An example where linear filtering techniques fail is the case of impulsive (long-tailed) noise filtering. Moreover, linear filters tend to blur edges and details of the signal. In such cases, nonlinear filtering would be preferable.

Median filter is one of the most popular nonlinear filters\cite{2}. It preserves edges of the signal while removing impulsive noise effectively. However, it often fails to provide sufficient smoothing of non-impulsive noise components.

In order to achieve various distributed noise removal, as well as the detail/edge-preservation, it is necessary to combine linear and nonlinear operations. One approach is the modified trimmed mean (MTM) filter\cite{3}, which selects the median from a moving signal window and averages only those points inside the window whose values are close to the median. Another approach is the FIR-median hybrid (FMH) filter\cite{4}, which is a class of median-type filters which contain FIR substructures. In the FMH filter, the input signal is filtered with separate FIR filters and the final output is the median of the outputs of the FIR filters.

We have also proposed a filter as the concept of combining linear and nonlinear operations. This is the mean and median hybrid (MMH) filter [5],[6]. In the MMH filter, the input signal is filtered by mean and median filter, separately. And then, the output of the MMH filters is selected from the output of the mean filter or the output of the median filter by using the difference between mean and median filter outputs. The motivation behind the choice of the difference two filter outputs is that in homogeneous regions, the two estimates will be close to each other, whereas in the presence of edges or impulses, they will differ considerably. Furthermore, MMH filters don’t need for a priori information on input signal. However, MMH filters don’t change their parameters even if a priori information can be obtained.

In this paper, the learning type of MMH (LMMH) filters are introduced. In the LMMH filter, mean and median filters of MMH filters are replaced with FIR and order statistics (OS) filters, respectively. Of course, the LMMH filter includes the MMH filter. The LMMH filter can be optimized by the least mean square (LMS) algorithm by using a priori information on input signal.

2. MMH FILTERS AND LEARNING TYPE OF MMH (LMMH) FILTERS
We have already proposed MMH filters in [5]. Because of MMH filters composing of mean and median filter, the coefficients of the each inner filter are fixed. LMMH filters which change mean and median filter of the inner filters of MMH filters for FIR and OS filter, respectively are introduced. The LMMH filter is the
generalization of the MMH filter. LMMH filters can get better performance than MMH filters by using LMS algorithm learning.

2.1 MMH Filters
The output of MMH filters is defined as
\[ y(i) = x_{\text{mea}}(i) + f[x_{\text{med}}(i) - x_{\text{mea}}(i)] \]  
where \( x_{\text{mea}}(i) \) and \( x_{\text{med}}(i) \) are given by
\[ x_{\text{med}}(i) = \text{MED}[x(i-M),...,x(i),...,x(i+M)] \]  
\[ x_{\text{mea}}(i) = \frac{1}{2N+1} \sum_{k=-N}^{N} x(i+k) \]
\( f[\cdot] \) is nonlinear function which is shown in Fig.2(a).

We explain the concept of the MMH filters briefly. In the calculating of the output of MMH filters, the test statistics used is the difference between the mean and the median estimate, which takes on large values in the presence of impulses or edges, indicating the median filter should be preferred as the output. In more homogeneous regions, the mean filter is chosen. MMH filters offer compromise between the sample median and the sample mean within the filter window.

2.2 LMMH Filters
The output of LMMH filters is defined as
\[ y(i) = x_{\text{mea}}(i) + f[x_{\text{med}}(i) - x_{\text{mea}}(i)] \]
where \( x_{\text{mea}}(i) \) and \( x_{\text{med}}(i) \) are given by
\[ x_{\text{med}}(i) = \sum_{j=1}^{2M+1} a_j x(j) = a^T x(i) \]  
\[ x_{\text{mea}}(i) = \sum_{k=-N}^{N} b_k x(i+k) = b^T x(i) \]
x_{\text{med}}(i) \( (j=1,\ldots,2M+1) \) are the order statistics of the \{x(i-M),...,x(i),...,x(i+M)\}. Therefore,
x_{\text{mea}}(i) \leq x_{\text{med}}(i) \leq \cdots \leq x_{(2M+1)}(i)
and
\[ x(i) = [x_{(1)}(i),x_{(2)}(i),...,x_{(2M+1)}(i)]^T \]
\[ x_{\text{med}}(i) = [x(i-N),...,x(i),...,x(i+N)]^T \]
a = [a_1,\ldots,a_{2M+1}]^T
b = [b_{-N},...,b_0,\ldots,b_N]^T
Since the mean and median filters are not always suitable for input signals, we replace mean and median filters of the MMH filter with FIR and OS filters, respectively. In LMMH filter, FIR and OS filters are able to be optimized by using a priori information of input signal.

![Fig.1 The structure of the MMH filter](image)

<table>
<thead>
<tr>
<th>Type</th>
<th>MMH</th>
<th>LMMH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fd'</td>
<td>Median</td>
<td>OS</td>
</tr>
<tr>
<td>Fd'</td>
<td>Mean</td>
<td>FIR</td>
</tr>
</tbody>
</table>

2.3 The Design Method of LMMH Filters
LMMH filters are the generalization of MMH filters. Therefore, if a priori information of input signal is given, the performance of LMMH filters can be improved by LMS algorithm, which leads to the minimization of mean square error (MSE). For optimizing LMMH filters under MSE criterion, the coefficients of OS and FIR filters and nonlinear function \( f[\cdot] \) are tuned by minimizing \( J(i) \) in eq.(7).

\[ J(i) = \frac{1}{2} (s(i) - y(i))^2 = \frac{1}{2} (e(i))^2 \]  
where \( s(i) \) is a original signal for training and \( e(i) \) is a estimation error.

The nonlinear function \( f[\cdot] \) shown in Fig.2(a) is defined by the two parameters \( \varepsilon \) and \( \zeta \). The optimization of \( \varepsilon \) and \( \zeta \) can be replaced with the optimization of two parameters \( \varepsilon \) and \( \theta \) (\( \zeta = \varepsilon + \theta \)) by using nonlinear function \( \mu(\cdot) \) (See Fig.2(b))

\[ f[x] = x \cdot \mu(x) \]  
Then, the coefficients of FIR and OS filters, the parameters of the nonlinear function are able to be optimized by following formula.

\[ a(i+1) = a(i) + K_a \cdot (\Delta y(i) / \Delta a(i)) \]  
\[ b(i+1) = b(i) + K_b \cdot (\Delta y(i) / \Delta b(i)) \]  
\[ e(i+1) = e(i) + K_e \cdot (\Delta y(i) / \Delta e(i)) \]  
\[ \theta(i+1) = \theta(i) + K_\theta \cdot (\Delta y(i) / \Delta \theta(i)) \]

where \( K_a, K_b, K_e, K_\theta \) are the step size and the coefficient vectors \( a(i) = [a_1(i),\ldots,a_{2M+1}(i)]^T \) and \( b(i) = [b_{-N}(i),\ldots,b_0(i),\ldots,b_N(i)]^T \) must be updated at each point \( i \), respectively. Here we denote derivatives of
$J(i) \text{ with respect to parameters in detail}$

$$\begin{align}
\frac{\partial J(i)}{\partial \alpha(i)} &= -\mu(i) \cdot \omega(i) \cdot \{1 - \mu(d(i)) - d(i) \cdot \frac{\partial \mu(d(i))}{\partial d(i)}\} \\
\frac{\partial J(i)}{\partial \beta(i)} &= -\varepsilon(i) \cdot \omega(i) \cdot \{\mu(d(i)) + d(i) \cdot \frac{\partial \mu(d(i))}{\partial d(i)}\} \\
\frac{\partial J(i)}{\partial \sigma(i)} &= -\varepsilon(i) \cdot d(i) \cdot \frac{\partial \mu(d(i))}{\partial \sigma(i)} \\
\frac{\partial J(i)}{\partial \sigma^2(i)} &= -\varepsilon(i) \cdot d(i) \cdot \frac{\partial \mu(d(i))}{\partial \sigma^2(i)}
\end{align}$$

where

$$d(i) = x_{\text{im}}(i) - x_{\text{os}}(i)$$

$$\mu(d(i)) = \begin{cases} 1 & \text{if } |d(i)| \leq \varepsilon(i) \\ 1 - \frac{|d(i) - \varepsilon(i)|}{\varepsilon(i)} & \text{if } \varepsilon(i) < |d(i)| < \varepsilon(i) + \theta(i) \\ 0 & \text{if } \varepsilon(i) + \theta(i) \leq |d(i)| \end{cases}$$

$$\frac{\partial \mu(d(i))}{\partial d(i)} = \begin{cases} -\frac{1}{\varepsilon(i)} & \text{if } \varepsilon(i) < |d(i)| < \varepsilon(i) + \theta(i) \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{\partial \mu(d(i))}{\partial \varepsilon(i)} = \begin{cases} \frac{1}{\varepsilon(i)} & \text{if } 0 < \varepsilon(i) < \varepsilon(i) + \theta(i) \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{\partial \mu(d(i))}{\partial \theta(i)} = \begin{cases} 0 & \text{if } \varepsilon(i) < |d(i)| < \varepsilon(i) + \theta(i) \\ 1 & \text{otherwise} \end{cases}$$

The initial values of the coefficients of FIR and OS filters are set as the coefficients of Wiener and optimal OS filters for training signal.

3. EXPERIMENTAL RESULTS

We will show practical effects on image processing of the LMMH filters. An image "LENA" is used in the simulations. The original image is corrupted by zero mean generalized exponential noise [7]. Consider the probability density function (pdf) of generalized exponential noise.

$$f_{\alpha}(x) = k e^{-\gamma |x|^\beta} ; |x| < \infty$$

where $\gamma$ and $\beta$ are positive, and where $k$ and $\gamma$ are obtained such that \[ k = (\beta \gamma^{1/\beta}) / 2 \Gamma(1/\beta) \]

with

$$\gamma = [\Gamma(3/\beta) / \Gamma(1/\beta)]^{1/\beta} \sigma^{-\beta}_n$$

where $\Gamma$ is the ordinary gamma function and $\sigma_n$ is the standard deviation. This pdf represents a broad range of noise behaviors ranging from very impulsive ($\beta < 1$), to shallow tailed ($\beta > 2$), approaching the uniform density as $\beta \to \infty$. The pdf includes Laplacian distribution ($\beta = 1$) and Gaussian distribution ($\beta = 2$) as special cases.

In the experiment, we use the added noises of various distribution $\beta$ ($\beta = 0.5, 0.75, 1.1, 1.5, 2, 3, 4$) and standard deviation $\sigma_n = 20$. Computer simulations have been carried out to compare the performance of the LMMH filters with MMH, Wiener, optimal OS filters. In the MMH and LMMH filters, the window size of mean and FIR filter is $5 \times 5$, the window size of median and OS filter is $3 \times 3$, respectively. The window size of Wiener filters and optimal OS filters are $5 \times 5$ and $3 \times 3$, respectively. In this simulation, LMMH filters have been designed by the image "LENA" corrupted by each distributed noise. The results of the simulation are presented in Fig.3. As can be seen, the LMMH filter has the best performance of all filters. Because of the LMMH filters being the compromise between FIR and OS filters, LMMH filters are useful for noise distribution ranging from the Gaussian to more highly "impulsive" distributions.

![Fig.3 The filter results of "LENA" with the generalized exponential noise](image)

Fig. 4(a) and 4(b) show the original image and image corrupted by noise ($\beta = 1, \sigma_n = 20$). The restored image by the proposed method compare with the Wiener filter, the optimal OS filter, and the MMH filter in Fig.4. The parameters of LMMH filter, Wiener filter and OS filter are listed in Table 2. As can be seen by comparing restored images, the LMMH filter give much better performance than the others. We can observe that the
optimal OS filter suppresses the noise efficiently while the Wiener filter preserves more signal details. The LMMH filter has the merits of the both filters.

![Original Image](image1) ![Noisy Image](image2)

![Wiener](image3) ![Optimal-OS](image4)

![MMH](image5) ![LMMH](image6)

**Fig. 4** Restored images

**Table 2 Parameters of each filter**

| (a) LMMH |  
|---|---|---|---|---|---|---|---|---|---|
| **FIR** | 0.0182 | 0.0296 | 0.0541 | 0.0413 | 0.0333 |  
| -0.0103 | -0.0465 | 0.130 | 0.00577 | 0.0112 |  
| -0.0194 | -0.000292 | 0.467 | -0.00213 | -0.0200 |  
| 0.584 | 0.00821 | 0.123 | -0.0497 | -0.00629 |  
| 0.0308 | 0.0364 | 0.0514 | 0.0270 | 0.00823 |  
| **OS** |  
| -0.047628 | 0.016029 | 0.035009 |  
| 0.301270 | 0.431882 | 0.295948 |  
| 0.086515 | 0.040471 | -0.041745 |  

Nonlinear function

\[
\varepsilon = -5.221651 \quad \theta = 20.955331
\]

(b) Wiener

| 0.00145 | 0.0129 | 0.0281 | 0.0212 | 0.0129 |  
| -0.00857 | 0.0352 | 0.130 | 0.0657 | 0.00301 |  
| -0.0119 | 0.0650 | 0.293 | 0.0639 | -0.0132 |  
| 0.00228 | 0.0685 | 0.129 | 0.0347 | -0.00637 |  
| 0.0135 | 0.0202 | 0.0271 | 0.0136 | -0.000891 |  

(c) Optimal-OS

| -0.012987 | 0.019421 | 0.019470 |  
| 0.267402 | 0.389370 | 0.254811 |  
| 0.057710 | 0.025910 | -0.021923 |  

4. CONCLUSIONS

In this paper, the learning type of MMH (LMMH) filters are introduced. In the LMMH filter, mean and median filters of MMH filters are replaced with FIR and OS filters, respectively. A procedure for designing an optimal LMMH filters by using a priori information on input signal under the MSE criterion has been developed. Experimental results demonstrated that LMMH filters can effectively remove from middle to long-tailed distributed noise.

REFERENCES


